FORM FACTOR OF THE PROCESS $\gamma^*\gamma^* \to \pi^\circ$ FOR SMALL VIRTUALITY OF ONE OF THE PHOTONS AND QCD SUM RULES

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ABSTRACT

We extend the QCD sum rule analysis of the $F_{\gamma^*\gamma^* \to \pi^\circ}(q_1^2, q_2^2)$ form factor into the region where one of the photons has small virtuality: $|q_1^2| \ll |q_2^2| \geq 1 \text{ GeV}^2$. In this kinematics, one should perform an additional factorization of short- and long-distance contributions. The extra long-distance sensitivity of the three-point amplitude is described by two-point correlators (bilocals), and the low-momentum dependence of the correlators involving composite operators of two lowest twists is extracted from auxiliary QCD sum rules. Our estimates for $F_{\gamma^*\gamma^* \to \pi^\circ}(q_1^2 = 0, q_2^2)$ are in good agreement with existing experimental data.

1. Introductory Remarks

In this work we present the result of our calculation [1,2] of the $F_{\gamma^*\gamma^* \to \pi^\circ}(q_1^2, q_2^2)$ form factor at small virtualities of one of the photons $|q_1^2| \ll |q_2^2| \geq 1$ GeV^2 within the QCD sum rule method [3]. In our first paper [1], we formulated a QCD sum rule approach to the problem and analyzed the structure of the OPE for the relevant three-point correlation function

$$\mathcal{F}_{\alpha\mu\nu}(q_1, q_2) = i \int e^{-iq_1 x - iq_2 y} \langle 0|T\left\{ J_\mu(x)\, J_\nu(y)\, j_\alpha^5(0) \right\}|0\rangle d^4x\, d^4y \;, \qquad (1)$$

with a particular emphasis on modifications required by the presence of the infrared (mass) singularities specific for the $q_1^2 \to 0$ limit. On Fig.1 we illustrate the schematic structure of the modified OPE for the three-point function Eq.(1).

The first row in Fig.1 corresponds to the usual operator expansion for the three-point correlation function. It was constructed [1] using the standard approach [4,5] valid in the kinematics when all the momentum invariants are large. In the $q_1^2 \to 0$ limit, this OPE is singular. In particular, the condensate terms explode like inverse powers of q_1^2. The perturbative term (the triangle loop) is formally finite in the small-q_1^2 limit, but it contains non-analytic contributions like $q_1^2 \ln\left(-q_1^2\right)$ and $q_1^4 \ln\left(-q_1^2\right)$. Just like in the pion FF case [6], because of these mass singularities, one should perform an additional factorization of short and long distance contributions (see also refs.[7]-[10]).
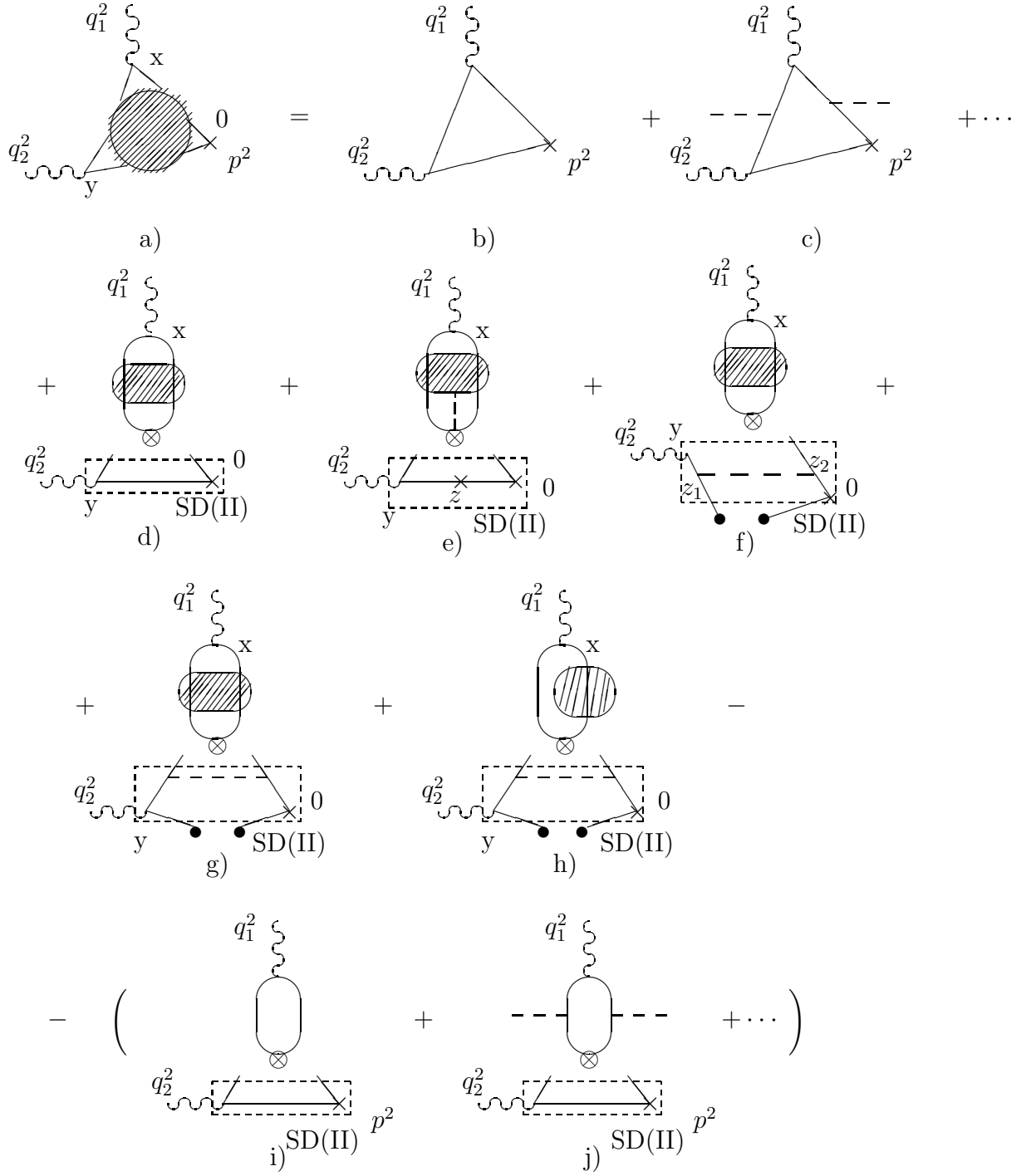

Fig.1

Schematically, the appropriate factorization procedure adds some extra terms into the original OPE. These terms are shown in the next three rows of Fig.1. The total contribution of terms staying inside the same bracket defines the so-called SD(I)-

regime for every graph of the first row. In this regime, all three currents are separated by short distances (cf. [6]), *i.e.*, all the intervals x^2, y^2 and $(x-y)^2$ are small. When q_1^2 goes to zero, they have the same singular behavior as the corresponding terms of the first row. Thus, in the OPE constructed for the essentially non-symmetric, small-q_1^2 kinematics [1], all the non-analytic terms mentioned above can be removed, and the final expression is regular in the $q_1^2 \to 0$ limit. The remaining terms in Fig.1 represent the SD(II)-regime, corresponding to a situation when the electromagnetic current $J_\mu(x)$ is separated by long distances from the two other currents, *i.e.*, the interval y^2 is small, while x^2 and $(x-y)^2$ are large.

The short-distance contributions are factorized into coefficient functions (CF), which, in our case (we do not consider radiative corrections), are given by a propagator or a product of propagators. In its turn, the long-distance contribution is represented by a two-point correlator (see [7,8,6]) of the electromagnetic current $J_\mu(x)$ and a composite operator of quark and gluonic fields denoted by $\otimes$ in Fig.1.

We emphasize that it is the twist[1] rather than dimension of the composite operators which determines the power behaviour of the contribution associated with these bilocals. This implies that, for a fixed twist, one should include the composite operators with an arbitrary number n of derivatives inside them.

By definition, the long-distance factor (the bilocal) accumulates nonperturbative information. One cannot calculate it in a straightforward perturbation theory. The idea is to incorporate again the QCD sum rule method. The starting point is a dispersion relation for the two-point correlator. As the next step, one should construct the OPE for this correlator in the region of large space-like q_1^2 and then analyze the resulting ("auxiliary") QCD sum rule to determine the parameters of the relevant model spectral density. In our case, they include the moments $\langle x^n \rangle_\rho$ of the ρ^0-meson wave functions $\varphi_\rho(x)$. To obtain the correlator at small q_1^2-values, one should just use this model spectral density in the original dispersion relation (see also refs. [6,9,10]).

2. QCD Sum Rule in the $|q_1^2| \ll |q_2^2| \geq 1$ GeV^2 kinematics

The final result for $\Phi_1(q^2, Q^2, M^2)$ — the theoretical part of the sum rule reads[2] ($q^2 \equiv -q_1^2, Q^2 \equiv -q_2^2$):

$$\Phi_1(q^2, Q^2, M^2) = \frac{\sqrt{2}\,\alpha_{e.m.}}{\pi} \frac{1}{M^2} \left\{ \int_0^1 dx\ e^{-Q^2 x/M^2 \bar{x}} \left\{ \left(1 + \frac{q^2 x}{M^2} e^{q^2 x/M^2}\right) + \right. \right.$$

$$+ e^{q^2 x/M^2} \left[\frac{2x}{M^2}\left(q^2 \ln \frac{(s_o + q^2)x}{M^2} - s_o\right) + \frac{x^2}{M^4}\left(q^4 \ln \frac{(s_o + q^2)x}{M^2} - q^2 s_o + \frac{s_o^2}{2}\right) \right] -$$

$$\left. \left. - \sum_{n=1}^{\infty} \left(\frac{q^2 x}{M^2}\right)^n \frac{\psi(n)(n+1)}{(n-1)!} \right\} + \right.$$

$$+\frac{\pi^2}{9}\langle\frac{\alpha_s}{\pi}GG\rangle\left[\frac{1}{2M^2Q^2}+\frac{1}{M^4}\int_0^1 dx\,\frac{x}{\bar{x}^2}\,e^{-Q^2x/M^2\bar{x}}\sum_{n=1}^\infty\frac{1}{n!}\left(\frac{q^2x}{M^2}\right)^{n-1}\right]+$$

$$+\frac{64\pi^3}{243}\,\alpha_s\langle\bar{q}q\rangle^2\,\frac{q^2}{Q^4M^2}\;+\;\frac{64\pi^3}{27}\,\alpha_s\langle\bar{q}q\rangle^2\,\frac{1}{2Q^2M^4}\;+$$

$$+\frac{4\pi^2}{3}\frac{f_\rho^V m_\rho}{m_\rho^2+q^2}\int_0^1 dx\frac{1}{\bar{x}^2M^2}\,e^{-Q^2x/M^2\bar{x}}\,e^{q^2x/M^2}$$

$$\times\left[-a_{V1}f_\rho^V m_\rho\left(\varphi_{\rho\perp}^{V1}(x)-\frac{4C_{V51}}{\bar{x}M^2}\varphi_{\rho\perp}^{V51}(x)\right)-f_\rho^A(1+2\bar{x})\left(\varphi_{\rho\perp}^A(x)-\frac{4C_{A5}}{\bar{x}M^2}\varphi_{\rho\perp}^{A5}(x)\right)\right]$$

$$+\frac{8\pi^2}{3}\frac{f_\rho^V m_\rho}{m_\rho^2+q^2}\int_0^1 d\alpha\,\alpha\int_0^1 d\beta\int_0^1[dx]_3\,e^{\,b/aM^2}$$

$$\times\left\{f_{3\rho}^A\varphi_{3\rho}^A(x_1,x_2;x_3)\left[\frac{c_1}{a^2M^2}-\frac{d_1}{2a^3M^4}\right]-f_{3\rho}^V\varphi_{3\rho}^V(x_1,x_2;x_3)\left[\frac{c_2}{a^2M^2}-\frac{d_2}{2a^3M^4}\right]\right\}$$

$$-\frac{64\pi^3}{27}\frac{\alpha_s\langle\bar{u}u\rangle}{M^4}\frac{m_\rho f_\rho^V f_\rho^T}{m_\rho^2+q^2}\iint_0^1 dxd\beta\,\frac{\beta\,\varphi_\rho^T(x)}{(1-x\beta)^3}\,e^{\beta(q^2x\bar{x}-Q^2x)/((1-x\beta)M^2)}\;-$$

$$-\frac{256\pi^3}{27}\frac{\alpha_s\langle\bar{u}u\rangle^2}{Q^2M^4}\iint_0^1 d\beta dy\,\frac{\beta(\bar{\beta}-\beta)y}{(1-y\beta)^3}\,e^{y\beta(q^2\bar{\beta}-Q^2)/((1-y\beta)M^2)}\;-$$

$$-\frac{256\pi^3}{27}\frac{\alpha_s\langle\bar{u}u\rangle}{Q^2M^4}\frac{m_\rho f_\rho^V f_\rho^T}{m_\rho^2+q^2}\frac{3}{4}q^2\times$$

$$\times\iiint_0^1 dxd\beta dy\,\frac{x\beta(\overline{x\beta}-x\beta)y}{(1-xy\beta)^3}\,\varphi_\rho^T(x)\,e^{yx\beta(q^2\overline{x\beta}-Q^2)/((1-yx\beta)M^2)}\bigg\}. \qquad (2)$$

where $f_\rho^V=0.2$ GeV, $m_\rho=0.77$ GeV ; the constants $f_\rho^A=-f_\rho^V m_\rho/4$, $a_{V1}=1/40$ are obtained from the equations of motion[2], the values $f_{3\rho}^A=0.6\cdot10^{-2}$ GeV^2, $f_{3\rho}^V=0.25\cdot10^{-2}$ GeV^2 are taken from the QCD sum rule estimates given in ref.[12].

Numerically most important contributions in Φ_1 come from: a) SD(I)-regime (first five rows of Eq.(2)) and b) ρ^o-meson contribution with leading twist wave functions (SD(II)-regime) in diagonal and nondiagonal correlators (see also [2]).

For the continuum threshold in the ρ-channel we take the standard value[3] $s_o\simeq1.5$ GeV^2 and asymptotic forms for the following ρ-meson wave functions [2,13,14]:

$$\varphi_{V1}=\varphi_{V1}^{as}=60x\bar{x}\,(2x-1),\quad\varphi_A=\varphi_A^{as}=6x\bar{x},$$
$$\varphi_{3A}=\varphi_{3A}^{as}=360x_1x_2x_3^2,\quad\varphi_{3V}=\varphi_{3V}^{as}=7!(x_1-x_2)x_1x_2x_3^2. \qquad (3)$$

The tensor wave function, however, appears in a non-diagonal correlator. For this reason, instead of $\varphi_T^{as} = 6x\bar{x}$, we use $\varphi_T = \frac{1}{2}(\delta(x) + \delta(\bar{x}))$ and take $f_\rho^T f_\rho^V m_\rho = -2\langle\bar{u}u\rangle$, which corresponds to the lowest-dimensional contribution to the nondiagonal correlator (see Fig.1,f).

The terms associated with three-particle twist-3 wave functions (Fig.1,e) are small (contribution of the order of a few percent). We expect that the twist-5 terms are also small. We took into account a specific type of power corrections, so-called contact terms, in a situation when they appear in the leading-twist bilocals. However, numerically the contact terms (see Fig.1,h) are small.

Finally, we write down the sum rule in the non-symmetric kinematics:

$$F_{\gamma^*\gamma^*\to\pi^\circ}(q^2, Q^2) = \frac{\sqrt{2}\alpha_{e.m.}}{\pi f_\pi}\left\{-2\int_{\sigma_o}^\infty d\sigma e^{-\sigma/M^2}\int_0^1 dx\frac{x\bar{x}(q^2x + Q^2\bar{x})^2}{[\sigma x\bar{x} + (q^2x + Q^2\bar{x})]^3}\right.$$
$$\left. + \Phi_1(q^2, Q^2, M^2)\frac{M^2\pi}{\sqrt{2}\,\alpha_{e.m.}}\right\}. \tag{4}$$

In Fig.2, c) we plot the $F_{\gamma^*\gamma^*\to\pi^\circ}(0, Q^2)$ form factor normalized by the value $F_{\gamma^*\gamma^*\to\pi^\circ}^{C.A.}(0, 0) = \sqrt{2}\alpha_{e.m.}/\pi f_\pi$[16]. We calculate it in the region $Q^2 \geq 1\text{GeV}^2$ and compare our results with experimental data reported by CELLO collaboration[11].
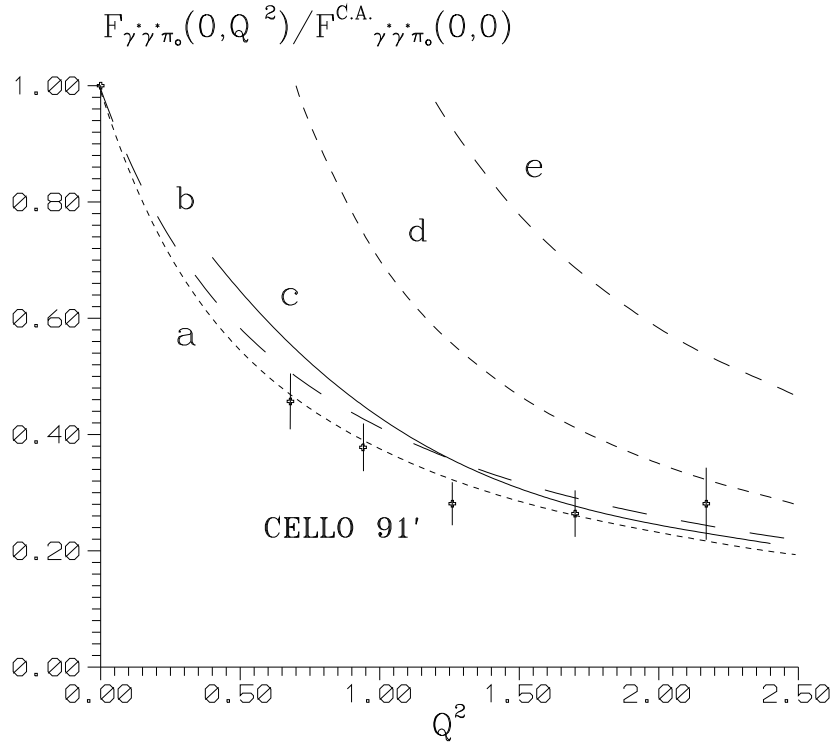
Fig.2

The scale σ_o, the continuum threshold in the pion channel was obtained by an explicit fitting procedure. The resulting values lie in the interval $0.6 \leq \sigma_o \leq 0.85\,\text{GeV}^2$,

i.e., they agree with existing estimates for the pion duality interval. The sum rule predictions are rather stable in the M^2-region $0.6\,\mathrm{GeV}^2 \leq M^2 \leq 1.3\,\mathrm{GeV}^2$ for different Q^2. Our results agree with experimental data within an accuracy of $15\% - 20\%$, usual for the QCD sum rules. The other curves prsented in Fig.2 correspond to: a) the vector dominance prediction, b) Brodsky–Lepage interpolation formula[15], d) leading twist pQCD-prediction using asymptotic form for pion wave function, e) leading twist pQCD-prediction using the Chernyak–Zhitnitsky form for the pion wave function.

Our sum rule Eq.(4) can be also used to calculate the form factor $F_{\gamma^*\gamma^*\to\pi^\circ}(q^2, Q^2)$ at small (but nonzero) momentum transfer $q^2 \leq m_\rho^2$ and fixed $Q^2 \geq 1\mathrm{GeV}^2$. However, there are no experimental data for this region. A detailed analysis of the sum rule including a detailed study of the sensitivity to various choices of the ρ-meson wave functions will be given elsewhere.

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